Short Papers

A Simple Analysis of Single- and Double-V-Groove Guides

Si-Fan Li, Zhong-Xiang Shen, and Xiao-Ming Lou

Abstract—**Single- and double-V-groove guides are analyzed by an approach based on a combination of the transverse resonance method and the numerical integration technique. Comparisons between the predicted results and available measured results for a single-V-groove guide show good agreement. The new approach is simple and accurate. Numerical results for the coupling characteristics of a double-V-groove guide are also presented.**

I. Introduction

Groove guide is potentially attractive as a low-loss waveguide in millimeter-wave and submillimeter-wave bands. Recently, this type of guide has attracted increasing attention because of its low loss, easy fabrication, large structural dimensions, and higher power handling capacity. V-groove guide has been proposed by Ho and Harris [1] as an alternative to rectangular-groove guide. Choi *et al.* [2] analyzed the single-groove guide by the conformal mapping technique. They pointed out that V-groove guide has propagation characteristics very similar to those of rectangular-groove guide, except that attenuation for the dominant mode is lower and rejection of higher order modes is more effective. So far the double-V-groove guide has not been investigated.

In this paper we propose a simple method for the analysis of single- and double-V-groove guides. The method is based on a combination of the transverse resonance approach [3] and the numerical integration technique [8]. The transverse equivalent networks for single- and double-V-groove guides are presented. The dispersion equations for the propagation characteristics of these waveguides are formulated by application of the transverse resonance condition. They are solved using numerical techniques. The approach used here is effectively simpler than the conformal mapping technique. Good agreement between our results for a single-V-groove guide and the experimental results in [2] confirms the validity of the proposed method. The analysis of a double-V-groove guide provides a theoretical basis for the design of V-groove guide directional couplers.

II. Theoretical Analysis

A. Single-V-Groove Guide

The cross section of the single-V-groove guide analyzed here is shown in Fig. 1. Fig. 2 shows a cross-sectional view of the electric field lines of the dominant mode present in the V-groove guide in Fig. 1. In order to analyze the V-groove guide, the transverse resonance approach [3] is employed. The parallel-plate waveguide region of the cross section is represented by uniform transmission lines. The V-groove is represented by a tapered line. The transverse equivalent network of a V-groove guide can be simplified by taking into account the symmetry of the waveguide structure. From Figs. 1 and 2, it is evident that at the plane $x = 0$ an electric wall can be introduced. The introduction of the electric wall at $x = 0$ leads to the simplified transverse equivalent network of the single-V-groove guide shown in Fig. 3. According to the method for the analysis of tapered lines used in [5], the following equation can be obtained:

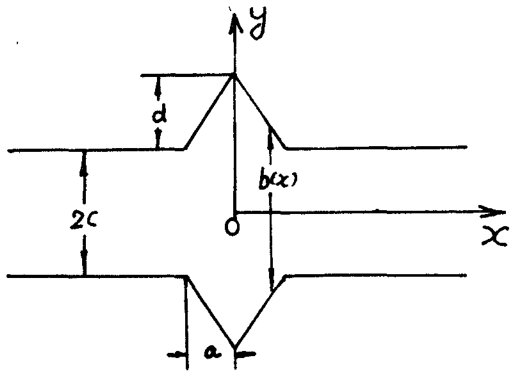

Fig. 1. Cross section of a single-V-groove guide.

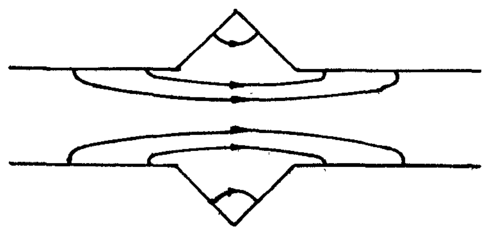

Fig. 2. Electric field lines in the cross section for the dominant V-groove guide.

$$\frac{dZ_{in}(x)}{dx} = -j\frac{K(x)}{Z_0(x)}Z_{in}^2(x) + jZ_0(x)K(x) \qquad (1)$$

where $Z_{in}(x)$ is the input impedance at x (see Fig. 3), and $Z_0(x)$ is the characteristic impedance of the tapered line at x, an expression for which was given in [3]. $K(x)$ is the propagation constant of the transmission line at x.

$$Z(x) = \frac{\omega\mu_0 K(x)}{K_c^2}$$

$$K^2(x) = K_c^2 - \left[\frac{\pi}{b(x)}\right]^2$$

$$b(x) = 2[c + d(1 - x/a)].$$

Here ω is the radian frequency, μ_0 is the permeability of free space, and K_c is the cutoff wavenumber of the dominant mode.

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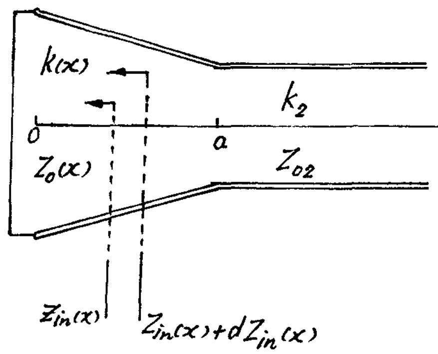

Fig. 3. Transverse equivalent circuit for the half cross section of a single-V-groove guide.

At $x = 0$ where the electric wall was introduced, the input impedance is equal to zero.

$$Z_{\text{in}}(x)|_{x=0} = 0. \tag{2}$$

Equation (1) can be solved using the fourth-order Runge–Kutta numerical technique [8]. The input impedance $Z_{\text{in}}(x)|_{x=a}$ can be calculated when K_c is assumed to be known.

By using the transverse resonance condition, the following equation can be obtained:

$$Z_{\text{in}}(x)|_{x=a} = -Z_{02} \tag{3}$$

where

$$Z_{02} = \frac{\omega\mu_0 K_2}{K_c^2}$$

$$K_2 = -j\sqrt{\left(\frac{\pi}{2c}\right)^2 - K_c^2}.$$

Let

$$Z_t(x) = \frac{K_c^2}{j\omega\mu_0} Z_{\text{in}}(x)$$

and substitute this equation into the above equations to yield

$$\begin{aligned} &\frac{dZ_t(x)}{dx} = Z_t^2(x) + K_c^2 - \left[\frac{\pi}{b(x)}\right]^2 \\ &Z_t(0) = 0 \\ &Z_t(a) = \sqrt{\left(\frac{\pi}{2c}\right)^2 - K_c^2}. \end{aligned} \tag{4}$$

The cutoff wavenumber, K_c, of the dominant mode in a single-V-groove guide can be calculated by solving eigenvalue problem (4).

B. Double-V-Groove Guide

The cross section of a double-V-groove guide is illustrated in Fig. 4. It is assumed that the structure is symmetrical with respect to the Y axis. The analysis can be simplified by applying odd- and even-mode excitation theory. When the odd mode is excited, the symmetrical plane forms an electric wall; when the even mode is excited, it is equivalent to a magnetic wall. The transverse equivalent circuit of a double-V-groove guide is shown in Fig. 5. Utilizing an approach similar to that introduced for the single-V-groove guide, the following two problems can be obtained:

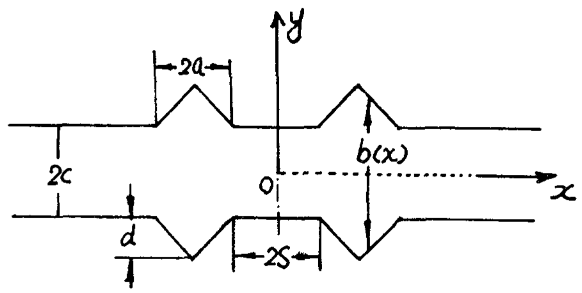

Fig. 4. Cross section a double-V-groove guide.

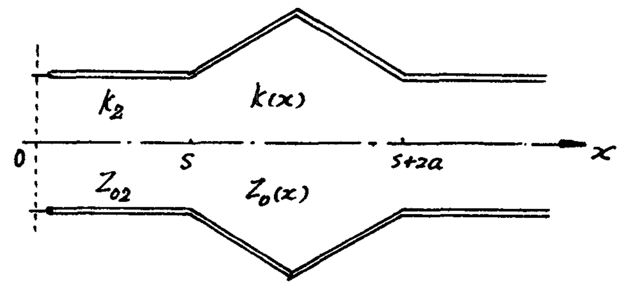

Fig. 5. Simplified transverse equivalent circuit for a double-V-groove guide.

TABLE I
GUIDE WAVELENGTHS AT $\lambda_0 = 3.117$ MM

Guide Dimensions 2c (mm)	a = d (mm)	Theoretical Results λ_g (mm) This Theory	Ref. [2]	Measured Results from [2] λ_g (mm)	Error Between Our Results and Those Measured in [2] (%)
	2	3.1544	3.155	3.156	0.05
10	4	3.1483	3.148	3.146	0.07
	6	3.1409	3.147	3.136	0.16
	2	3.1432	3.144	3.146	0.09
12	4	3.1403	3.139	3.138	0.07
	6	3.1360	3.138	3.130	0.19

$$\text{odd mode}\begin{cases} \dfrac{dZ_{to}(x)}{dx} = Z_{to}^2(x) + K_{co}^2 - \left[\dfrac{\pi}{b(x)}\right]^2 \\ Z_{to}(s+2a) = P \\ Z_{to}(s) = -P\cdot\tanh(s\cdot P) \end{cases} \tag{5}$$

$$\text{even mode}\begin{cases} \dfrac{dZ_{te}(x)}{dx} = Z_{te}^2(x) + K_{ce}^2 - \left[\dfrac{\pi}{b(x)}\right]^2 \\ Z_{te}(s+2a) = P \\ Z_{te}(s) = -P/\tanh(s\cdot P) \end{cases} \tag{6}$$

where

$$P = \sqrt{\left(\frac{\pi}{2c}\right)^2 - K_{co(e)}^2}$$

$$b(x) = \begin{cases} c + d(x-s)/a & (s < x < s+a) \\ C + d(2a+s-x)/a & (s+a < x < s+2a). \end{cases}$$

TABLE II
GUIDE WAVELENGTHS AT X BAND

Frequencies f (GHz)	Predicted Values λ_g (cm) Ours	Predicted Values [2] Ref. [2]	Measured Values [2] λ_g (cm)	Error Between Our Values and Those Measured in [2] (%)
8.02	3.8505	3.855	4.000	3.75
8.53	3.6078	3.611	3.704	2.60
9.15	3.3519	3.355	3.448	2.12
9.695	3.1567	3.157	3.226	2.15
10.25	2.9785	2.980	3.030	1.70
10.81	2.8192	2.820	2.857	1.33
11.375	2.6750	2.676	2.703	1.04
11.97	2.5385	2.539	2.564	1.00
12.26	2.4998	2.501	2.500	0.01

Guide dimensions: $2a = 3$ cm, $d = 4.5$ cm, $2c = 7.5$ cm.

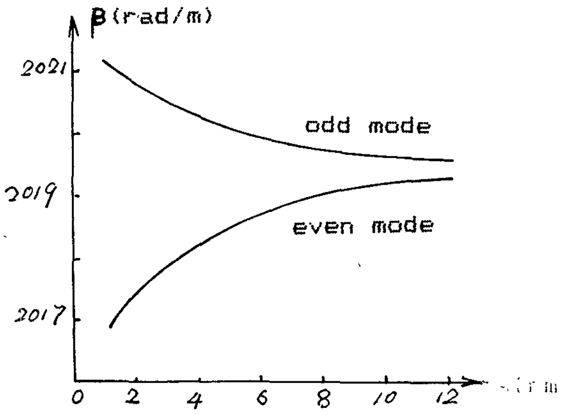

Fig. 6. Propagation constants of odd and even modes in a double-V-groove guide versus groove separation s ($2c = 10$ mm, $a = 5$ mm, $d = 2.5$ mm, $\lambda_0 = 3.08$ mm).

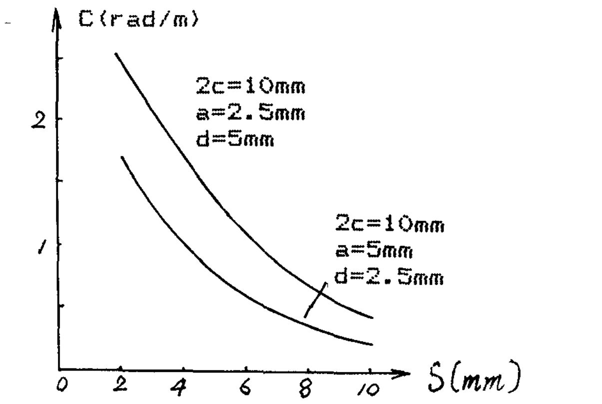

Fig. 7. Coupling coefficient C versus groove separation s ($\lambda_0 = 3.08$ mm, $2c = 10$ mm).

As in the case of single-V-groove guide, (5) and (6) can be solved by the Runge–Kutta technique [8], and the cutoff wavenumbers for odd and even modes in a double-V-groove guide can be calculated.

III. Numerical Results

In order to verify the method proposed here, the theory was first applied to a single-V-groove guide. The calculated results for the guide wavelengths of the dominant mode at 100 GHz and X band are shown in Tables I and II along with the calculated and measured values given in [2]. The agreement is excellent; thus the accuracy of the present method has been verified by other theoretical and experimental results. The reason why the error between the predicted and measured guide wavelengths is significantly higher at X band than at 100 GHz is that the accuracy of measurement at X band is low [2]. It is probable that similar accuracy can be obtained for a double-V-groove guide. From (5) and (6), the cutoff wavelengths and the propagation constants of odd and even modes in a double-V-groove guide can be obtained. Fig. 6 shows the variation of the propagation constants of the odd and even modes with respect to groove separation.

The coupling coefficient between two grooves is defined as follows:

$$C = 10\log_{10}\frac{\beta_o - \beta_e}{\beta_o + \beta_e} \tag{7}$$

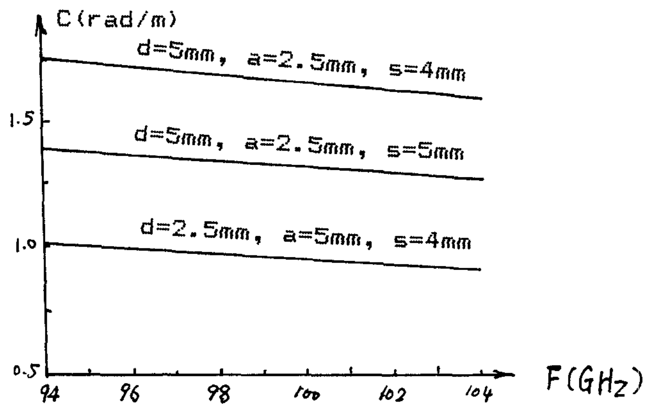

Fig. 8. Coupling coefficient versus frequency ($2c = 10$ mm).

where β_o and β_e are the propagation constants of the odd and even modes respectively. The variation of the coupling coefficient with groove separation is shown in Fig. 7, from which s can be determined if C and other parameters are given. The frequency characteristics of the coupling coefficient C are shown in Fig. 8. It can be seen that the variation of the coupling coefficient with respect to frequency is small. Therefore, the double-V-groove guide is a potential structure for broad-band directional couplers in the millimeter-wave band.

IV. Conclusions

The transverse resonance approach associated with the Runge–Kutta numerical integration technique has been employed to calculate the propagation characteristics of the dominant mode in single-V-groove guide. Numerical results for the guide wavelength of the dominant mode in a single-V-groove guide have been obtained and shown to agree well with available results in the literature. The analysis of a double-V-groove guide has also been performed. Although the method described in this paper is applied to the analysis of the V-shaped groove guide, the analysis can be extended to quite general shapes such as semicircular grooves, trapezoidal grooves, and closed V-groove guides.

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Correct Determination of TE and TM Cutoff Wavenumbers in Transmission Lines with Circular Outer Conductors and Eccentric Circular Inner Conductors

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***Abstract*—The cutoff wavenumbers of TE and TM modes (higher order modes) in transmission lines with circular outer conductors and eccentric circular inner conductors are carefully evaluated. The correctness of Kuttler's bounds is confirmed and the reason why some of the values obtained lie outside the bounds and some of the modes could not be found in Vishen's paper is given. A reliable technique for accurately determining the roots of an analytical function is proposed for finding cutoff wavenumbers in such a way as to avoid missing any modes.**

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I. Introduction

Calculation of the cutoff wavenumbers of TE and TM modes in transmission lines having a circular outer conductor and an eccentric circular inner conductor has been of great interest to many authors. By means of conformal mapping combined with intermediate methods for the lower bounds and the Rayleigh–Ritz method for the upper bounds, these cutoff wavenumbers were evaluated by Kuttler [1] for several different relative dimensions of the structure. A special analytical shape perturbation method [2] was developed by Roumeliotis *et al.* for treating small eccentricities. Vishen *et al.* [3] employed a method which may be conveniently used for evaluating the cutoff wavenumbers in the structure for large or small eccentricities and different radius ratios, and the examples in [1] were repeated. However, some of the results computed by Vishen obviously contradict Kuttler's bounds. As we can see from [4], some confusion still exists.

In this paper, the technique used in [3] is employed. Deficiencies of the formulation in [3] are pointed out, and by careful derivation, a new expression is obtained. Cutoff wavenumbers of the structure for all the cases considered in [3] are carefully evaluated. All of our results lie in the bounds reported by Kuttler and are quite close to the upper bounds. The reason why some of the modes could not be found and some of the values fall outside Kuttler's bounds in [3] is given.

Like the analytical technique used here, many other methods, among them the method of moments, also reduce cutoff wavenumbers to the zeros of an analytical function. Hence, correctly determining the zeros of an analytical function is a problem of general interest. Some iterative algorithms, e.g. Muller iteration, are frequently employed [5] for tackling the problem. To the best of the authors' knowledge, it is difficult to avoid missing roots by simply using such iterative algorithms, because one rarely knows how many zeros exist inside the given frequency band. In this paper, a new technique is developed on the basis of a combination of the contour integral method [6] with Muller iteration. The method exhibits accuracy and efficiency, as well as reliability.

II. Results and Discussion

We use the method employed in [3], i.e., separation of variables and the use of addition theorems for Bessel functions to satisfy the boundary condition at the outer circular conductor. This method, as pointed out by certain authors [4], is not new and can be found in many papers. However, with the matrix elements expressed in ratio forms of Bessel functions, the formulas presented in [3] have deficiencies which may sometimes result in mistakes. By a careful derivation (see the Appendix) we obtain, for TE modes,

$$\det\left[P_{nm}(k)\right]=0 \tag{1}$$

where the elements of the determinant are given by

$$P_{nm}(k)=\left[J_n'(kb)Y_m'(ka)-Y_n'(kb)J_m'(ka)\right] \cdot\left[J_{n-m}(kd)+(-1)^l J_{n+m}(kd)\right] \tag{2}$$